[07-09-09A-T10]

Absolute value equations

It is important to remember that absolute value is always a distance and therefore always a positive number. This immediately implies that eliminating the absolute value bars *formally* amounts to the following:

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

Providing this is understood and remembered, the following theorems simplify working with absolute value.

■ **Theorem 1.** $|x| = k \Longleftrightarrow x = \pm k$

■ **Theorem 2.** $|ax + b| = k \Longleftrightarrow ax + b = \pm k$

■ **Theorem 3.** $|x| = |a| \Longrightarrow x = \pm a$

■ **Theorem 4.** $|ax + b| = |cx + d| \Longrightarrow ax + b = \pm(cx + d)$

■ **[EX1]**

Find all numbers x such that $|x| = 5$.

Solution.

$|x| = 5 \Longrightarrow x = \pm 5$, by theorem 1.

■ **[EX2]**

Find all numbers x such that $|3x - 9| = 5$.

Solution.

$|3x - 9| = 5 \Longrightarrow 3x - 9 = \pm 5$

Then

$3x = 9 \pm 5$

$x = \frac{9 \pm 5}{3}$

equivalently

$x = \frac{14}{3}$ or $x = \frac{4}{3}$.

Or, just write $x \in \{\frac{14}{3}, \frac{4}{3}\}$

■ **[EX3]**

Find all numbers x such that $|5(x+2)-7| = -6$.

Solution.

$$|5(x+2)-7| = -6 \Longleftrightarrow |5x+3| = 6$$

Then

$$5x + 3 = \pm 6$$

$$x = \frac{\pm 6 - 3}{5}$$

$x = \frac{3}{5}$ or $x = -3$.